Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

3187654992

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 Marks)

1 A sports car of mass 1.2 tonnes is being tested on a horizontal, straight section of road. After t s, the car has travelled x m from the starting line and its velocity is $v\,\mathrm{m\,s^{-1}}$. The engine produces a driving force of 4000 N and the total resistance to the motion of the car is given by $\frac{40}{49}v^2$ N. The car crosses the starting line with speed $10\,\mathrm{m\,s^{-1}}$.

(i) Write down an equation of motion for the car and solve it to show that $v^2 = 4900 - 4800\,\mathrm{e}^{-\frac{1}{735}x}$. **[7]**

(ii) Hence find the work done against the resistance to motion over the first 100 m beyond the starting line. **[5]**

2 On a building site, a pulley system is used for moving small amounts of material up to roof level. A light pulley, which can rotate freely, is attached with its axis horizontal to the top of some scaffolding. A light inextensible rope hangs over the pulley with a counterweight of mass m_1 kg attached to one end. Attached to the other end of the rope is a bag of negligible mass into which m_2 kg of roof tiles are placed, where $m_2 < m_1$. This situation is shown in Fig. 2.

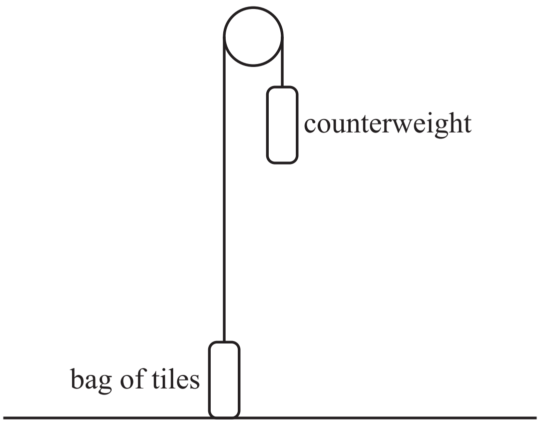

Fig. 2

Initially the system is held at rest with the rope taut, the counterweight at the top of the scaffolding and the bag of tiles on the ground. When the counterweight is released, the bag ascends towards the top of the scaffolding. At time t s the velocity of the counterweight is $v\,\mathrm{m\,s^{-1}}$ downwards.

The counterweight is made from a bag of negligible mass filled with sand. At the moment the counterweight is released, this bag is accidentally ripped and after this time the sand drops out at a constant rate of $\lambda\,\mathrm{kg\,s^{-1}}$.

(i) Find the equation of motion for the counterweight while it still contains sand, and hence show that

$$v = gt + \frac{2gm_2}{\lambda}\ln\left(1 - \frac{\lambda t}{m_1 + m_2}\right).$$ **[8]**

(ii) Given that the sand would run out after 10 seconds and that $m_2 = \frac{4}{5}m_1$, find the maximum velocity attained by the counterweight towards the ground. You may assume that the scaffolding is sufficiently high that the counterweight does not hit the ground before this velocity is reached. **[4]**

Section B (48 Marks)

3 A uniform rigid rod AB of mass m and length $2a$ is freely hinged to a horizontal floor at A. The end B is attached to a light elastic string of modulus λ and natural length $5a$. The other end of the string is attached to a small, light, smooth ring C which can slide along a horizontal rail. The rail is a distance $7a$ above the floor and C is always vertically above B. The angle that AB makes with the floor is θ. The system is shown in Fig. 3.

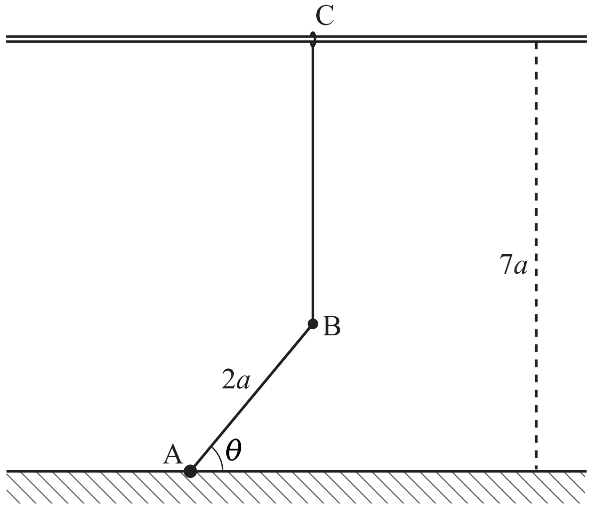

Fig. 3

(i) Find the potential energy, V, of the system and hence show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = a\cos\theta\left(mg - \frac{4\lambda}{5}(1-\sin\theta)\right).$$ **[6]**

(ii) Show that there is a position of equilibrium when $\theta = \frac{1}{2}\pi$ and determine whether or not it is stable. **[6]**

There are two further positions of equilibrium when $0 < \theta < \pi$.

(iii) Find the magnitude of the tension in the string and the vertical force of the hinge on the rod in these positions. **[3]**

(iv) Show that $\lambda > \frac{5mg}{4}$. **[4]**

(v) Show that these positions of equilibrium are stable. **[4]**

4 **(a)** A pulley consists of a central cylinder of wood and an outer ring of steel. The density of the wood is $700\,\text{kg m}^{-3}$ and the density of the steel is $7800\,\text{kg m}^{-3}$. The pulley has a radius of 20 cm and is 10 cm thick (see Fig. 4.1).

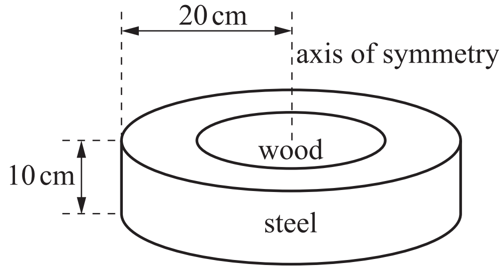

Fig. 4.1

Find the radius that the central cylinder must have in order that the moment of inertia of the pulley about the axis of symmetry shown in Fig. 4.1 is $1.5\,\text{kg m}^2$. **[6]**

(b) Two blocks P and Q of masses 10 kg and 20 kg are connected by a light inextensible string. The string passes over a heavy rough pulley of radius 25 cm. The pulley can rotate freely and the string does not slip. Block P is held at rest in smooth contact with a plane inclined at 30° to the horizontal, and block Q is at rest below the pulley (see Fig. 4.2).

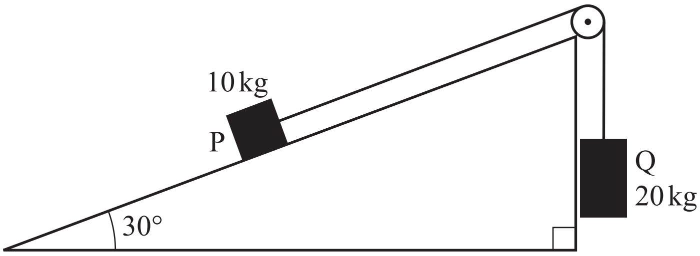

Fig. 4.2

At t s after the system is released from rest, the pulley has angular velocity ω rad s^{-1} and block P has constant acceleration of 2 m s^{-2} up the slope.

(i) Show that the net loss of energy of the two blocks in the first t seconds of motion is $87t^2$ J and use the principle of conservation of energy to show that the moment of inertia of the pulley about its axis of rotation is $\frac{87}{32}$ kg m^2. **[6]**

When $t=3$ a resistive couple is applied to the pulley. This resistive couple has magnitude $(2\omega + k)$ N m, where k is a constant. The couple on the pulley due to tensions in the sections of string is $\left(\dfrac{147}{4} - \dfrac{15}{8}\dfrac{d\omega}{dt}\right)$ N m in the direction of positive ω.

(ii) Write down a first order differential equation for ω when $t \geqslant 3$ and show by integration that

$$\omega = \frac{1}{8}\left((45+4k)e^{\frac{64}{147}(3-t)} + 147 - 4k\right).$$ **[8]**

(iii) By considering the equation given in part **(ii)**, find the value or set of values of k for which the pulley

(A) continues to rotate with constant angular velocity,

(B) rotates with decreasing angular velocity without coming to rest,

(C) rotates with decreasing angular velocity and comes to rest if there is sufficient distance between P and the pulley. **[5]**

END OF QUESTION PAPER

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4764/01 Mechanics 4

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (24 marks)

1 (i)

1 (ii)

2 (i)	
	(answer space continued on next page)

2 (i)	**(continued)**
2 (ii)	

Section B (48 marks)

3 (i)	

3 (ii)	
3 (iii)	

3 (iv)	
3 (v)	
	(answer space continued on next page)

3 (v)	**(continued)**
4 (a)	

4 (b)(i)	

4 (b)(ii)	

4(b)(iii)	

OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4764**: Mechanics 4

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	**Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$1200v\frac{dv}{dx} = 4000 - \frac{40}{49}v^2$	M1	For use of N2L with any expression for *a* –numerical/sign slips only
				A1	Correct, with any expression for *a*
			$\int \frac{1470v}{4900 - v^2} dv = \int dx$	M1*	Separate variables leading to $A\ln(B - Dv^2) = Ex(+c)$
			$-\frac{1470}{2}\ln\left|4900 - v^2\right| = x + c$	A1ft	Integrate, condone missing modulus signs and + *c*. FT their DE
			When $x = 0$, $v = 10$, so $c = -735 \ln 4800$	M1dep*	Use correct initial condition leading to $c = \ldots$
			$v^2 = 4900 - 4800e^{-\frac{1}{735}x}$	M1	Using correct log laws (not trivialised) to make v^2 the subject
				E1	CAO
				[7]	

Question			Answer	Marks	Guidance
1	(ii)		$F = \frac{40}{49}v^2$		
			so $F = \frac{40}{49} \times \; 4900 - 4800e^{-\frac{1}{735}x}$	B1	
			Work done = $\int F\mathrm{d}x$	M1	Integral of their F
			$= \frac{40}{49}\left(4900x + 3528000e^{-\frac{1}{735}x}\right)$	A1	CAO (AEF)
			$= \frac{40}{49}\left[4900x + 3528000e^{-\frac{1}{735}x}\right]_0^{100}$	M1	Integrated expression evaluated between both correct limits provided integrated expression is of the form $Ax + Be^{-\frac{1}{735}x}$
			= 33649.98057… = 33.6 kJ to 3 sf	A1	CAO
				[5]	
		OR	Total work done by driving force = 400000 J	B1	
			Change in KE =	M1	
			$\frac{1}{2} \times 1200 \times \; 4900 - 4800e^{-\frac{100}{735}} \; - \frac{1}{2} \times 1200 \times 10^2$		
			= 366350.0194…	A1	
			Work done against resistance = total work done by driving force – change in kinetic energy	M1	
			So work done against resistance = 400000 – 366350.01… = 33649.98… = 33.6 kJ to 3 sf	A1	CAO

Question			Answer	Marks	Guidance
2	(i)		mass of counterweight = $m_1 - \lambda t$	B1	Condone use of δt
			$(m_1 - \lambda t)\dfrac{dv}{dt} = (m_1 - \lambda t)g - T$	M1*	Equation of motion of the counterweight – their $(m_1 - \lambda t)$ - must be three terms – condone sign errors (condone use of m_1 for $(m_1 - \lambda t)$ for this M mark only)
			$m_2\dfrac{dv}{dt} = T - m_2 g$	M1*	Equation of motion of the bag – 3 terms – condone sign errors
			$(m_1 - \lambda t + m_2)\dfrac{dv}{dt} = (m_1 - \lambda t - m_2)g$	M1 dep*	Eliminate T; must be using $(m_1 - \lambda t)$
		OR	For the first five marks $(m_1 - \lambda t)g - m_2 g = (m_1 - \lambda t + m_2)\dfrac{dv}{dt}$		CAO Scores the first five marks (any v terms M0 M0) B1 as above M1 3 terms shown without $\dfrac{dv}{dt}$ – condone sign errors M1 3 terms shown with $\dfrac{dv}{dt}$ – condone sign errors A1 A1 each side must have scored both M marks
			$\int dv = g\int \dfrac{m_1 - \lambda t - m_2}{m_1 - \lambda t + m_2}dt$	M1	Separating the variables and integrating to obtain $v = gt + A\ln(\text{must include } m_1, m_2 \text{ and } \lambda t)$
			$v = g\int 1 - \dfrac{2m_2}{m_1 - \lambda t + m_2}dt$		
			$= gt + \dfrac{2m_2 g}{\lambda}\ln\lvert m_1 - \lambda t + m_2\rvert + c$	A1	Condone missing modulus signs throughout
			at $t = 0$, $v = 0$, so $c = -\dfrac{2m_2 g}{\lambda}\ln\lvert m_1 + m_2\rvert$		
			$v = gt + \dfrac{2m_2 g}{\lambda}\ln\left\lvert\dfrac{m_1 - \lambda t + m_2}{m_1 + m_2}\right\rvert$		
			$= gt + \dfrac{2m_2 g}{\lambda}\ln\left\lvert 1 - \dfrac{\lambda t}{m_1 + m_2}\right\rvert$	E1	
				[8]	

Question			Answer	Marks	Guidance
2	(ii)		$\lambda = \frac{1}{10} m_1$ $t = \dfrac{m_1 - m_2}{\lambda}$ = 2 seconds $v = 1.13162\ldots$ = 1.13 m s^{-1}	B1 M1* M1dep* A1 **[4]**	Seen Setting their $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$ or other argument and obtaining t Substitute their t and $\lambda = \frac{1}{10} m_1$ into v $2g + 16g \ln(8/9)$
3	(i)		$V = mga\sin\theta$ (+ c) $+\dfrac{\lambda}{2(5a)}\ 2a - 2a\sin\theta\ ^2$ Hence $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = mga\cos\theta$ $+\dfrac{\lambda}{10a} \times 2\ \ 2a - 2a\sin\theta\ \times\ -2a\cos\theta$ $= a\cos\theta\left(mg - \dfrac{4\lambda}{5}\ 1 - \sin\theta\ \right)$	B1 M1 A1 M1 A1 E1 **[6]**	GPE (Allow any level for GPE = 0) EPE using $\dfrac{\lambda x^2}{2a}$ with $5a$ for the natural length and genuine attempt at the extension Correct (AEF) e.g. $mga\sin\theta + \dfrac{2\lambda a}{5}(1 - 2\sin\theta + \sin^2\theta)$ Differentiate their V (at least one of their terms correctly differentiated – V must be GPE + EPE) AEF e.g. $mga\cos\theta + \dfrac{2\lambda a}{5}(-4\cos\theta + 2\sin\theta\cos\theta)$ AG

Question			Answer	Marks	Guidance
3	(ii)		$\theta = \frac{1}{2}\pi \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$	M1	Or $V' = 0 \therefore a\cos\theta = 0 \Rightarrow \theta = \pi/2$
			Hence, equilibrium	E1	
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -a\sin\theta\left(mg - \frac{4\lambda}{5}\ 1-\sin\theta\ \right)$	M1	Differentiate
			$+a\cos\theta\left(\frac{4\lambda}{5}\cos\theta\right)$	A1	Correct
			When $\theta = \frac{1}{2}\pi$		
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -a(mg) + 0 < 0$	M1	Substitute $\theta = \frac{1}{2}\pi$ into their $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$
			Therefore, unstable	E1 **[6]**	Must establish < 0
3	(iii)		Taking moments about A	M1	Or use $T = \frac{\lambda x}{l_0}$ with their x and $5mg = 4\lambda(1-\sin\theta)$
			$mga\cos\theta = 2aT\cos\theta \Rightarrow T = \frac{1}{2}mg$,	A1	
			Resolving vertically $R = \frac{1}{2}mg$	A1ft **[3]**	For ft must be in terms of m and g only
3	(iv)		$mg - \frac{4\lambda}{5}\ 1-\sin\theta\ = 0$	B1	
			$\Rightarrow \sin\theta = \frac{4\lambda - 5mg}{4\lambda}$	B1	AEF – making $\sin\theta$ the subject or a form that allows consideration of values of $\sin\theta$ e.g. $1-\sin\theta = ...$
			Since $0 < \theta < \pi$, $\sin\theta > 0$	M1	Consideration of $\sin\theta$ in the interval $0 < \theta < \pi$ (maybe implicit) so allow no justification given for sin $\theta > 0$ in the interval.
			$\Rightarrow \lambda > \frac{5mg}{4}$	E1 **[4]**	Must have considered why $\sin\theta > 0$ or relative position of B with respect to A

Question			Answer	Marks	Guidance
3	(v)		$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = a\left(\frac{4\lambda}{5}\left(1+\sin\theta-2\sin^2\theta\right)-mg\sin\theta\right)$	M1*	consider second differential of V and substitute $\sin\theta = 1-\frac{5mg}{4\lambda}$
			$= a\left(\frac{4\lambda}{5}\left(1+\frac{4\lambda-5mg}{4\lambda}-2\left(\frac{4\lambda-5mg}{4\lambda}\right)^2\right)-mg\left(\frac{4\lambda-5mg}{4\lambda}\right)\right)$	A1	…in terms of λ, m only CAO in terms of λ, m only (AEF)
			$= \frac{mga}{4\lambda}\left(8\lambda-5mg\right) > 0$	M1 dep*	consider sign using result from (iv) and their $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$ in terms of λ, m only
			because $\lambda > \frac{5mg}{4}$, therefore stable	E1	
		OR		M1*	Substituting $mg - \frac{4\lambda}{5}(1-\sin\theta)$ in V'' and obtaining V'' in terms of $\cos^2\theta$ only
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{4\lambda}{5}a\cos^2\theta$	A1	CAO
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} > 0$ in the interval $0 < \theta < \pi$…	M1 dep*	Considering sign of their $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{4\lambda}{5}a\cos^2\theta$ for $0 < \theta < \pi$
			…except at $\theta = \frac{\pi}{2}$ but this is the other equilibrium position.	E1 **[4]**	Stable – with reference to $\theta \neq \frac{\pi}{2}$

Question			Answer	Marks	Guidance
4	(a)		$M_{WOOD} = \pi r^2(0.1)(700)$ and $M_{STEEL} = \pi(0.2)^2(0.1)(7800) - \pi r^2(0.1)(7800)$	B1	Both masses - maybe implicit in an integral – condone working in centimetres
			$I_{\text{WOOD}} = 35\pi r^4$	B1	$0.5(70\pi r^2)r^2$ - condone working in centimetres
			$I_{\text{STEEL}} = \frac{78}{125}\pi - 390\pi r^4$	M1*	Consider difference of two Is – must be of the form $A - Br^4$ - allow derivation of $0.5M(R^2 + r^2)$ from first principles (where M is the mass of the annulus) or $0.5(\text{their } M_{STEEL})(r^2 + 0.2^2)$.
				A1	$0.5\pi(31.2)(0.2)^2 - 0.5\pi(780r^2)r^2$
			$\frac{78}{125}\pi - 390\pi r^4 + 35\pi r^4 = 1.5$	M1 dep*	$I_{STEEL} + I_{WOOD} = 1.5$
			so r = 14.3 cm to 3 sf	A1	
				[6]	
4	(b)	(i)	$\frac{1}{2}\times 10\times v^2 + \frac{1}{2}\times 20\times v^2 + \ldots$	B1*	KE for one (allow centimetres/metres)
			$\ldots 10g\times x\sin 30 - 20g\times x$	B1*	GPE for one (allow centimetres/metres)
			$a = 2$, $v = 2t$, $s = t^2$	M1 dep*	velocity and distance travelled in terms of a single variable
			$= \frac{1}{2}\times 10\times (2t)^2 + \frac{1}{2}\times 20\times (2t)^2 + \ldots$		
			$\ldots 10g\times t^2\sin 30 - 20g\times t^2$		
			$= 87t^2$	E1	AG
			$r\omega = 2t$ with $r = 0.25$ so $\omega = 8t$ and $87t^2 = \frac{1}{2}I(8t)^2$	M1	GPE, KE and RKE terms with conservation of energy condone numerical slips
			Hence $I = \frac{87}{32}$	E1	
				[6]	

Question			Answer	Marks	Guidance
4	**(b)**	**(ii)**	$36.75-1.875\dot{\omega}\ -(2\omega+k)=\frac{87}{32}\dot{\omega}$	M1 A1	Use $C=\ I\dot{\omega}$ correct number of terms CAO
			$147\int\frac{1}{64\omega+32k-1176}\,\mathrm{d}\omega=\int -\mathrm{d}t$	M1	Separate variables (this mark is not dependent on the previous M mark) or using a correct integrating factor for their DE
			$\frac{147}{64}\ln\lvert 64\omega+32k-1176\rvert=-t(+c)$	M1*	Integrate to $A\ln(B+D\omega)=Et(+c)$ or equivalent if using IF approach
				A1	CAO and condone not using modulus
			When t = 3, $\omega = 24$	B1 dep*	Use condition
			so c = $\frac{147}{64}\ln\lvert 32k+360\rvert$ + 3	A1	Correctly find c
			$\omega=\frac{1}{8}\quad 45+4k\quad e^{\frac{64}{147}(3-t)}+147-4k$	E1	
				[8]	
4	**(b)**	**(iii)**			
		(A)	Critical value for 45 + 4k = 0 k = –11.25	M1 A1	The expression *45+ 4k* seen in working in part (A)
			Critical value for 147 – 4k = 0	M1	May be awarded in (B) or (C) – the expression $147-4k$ seen in (B) or (C) only
		(B)	*their* $-11.25<k\le 36.75$	F1	
		(C)	k > 36.75	A1	
				[5]	

4764 Mechanics 4

General Comments:

The work on this unit was generally of a good standard. Many of the candidates were very competent and these demonstrated a sound understanding of the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident when solving differential equations and manipulating complicated expressions and most demonstrated a solid knowledge of the techniques and concepts required. The majority of candidates appeared to have sufficient time to complete the paper. The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions:

Question No. 1 – Variable force

In part (i) the vast majority of candidates correctly applied Newton's second law of motion and used $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for the acceleration. The technique of separation of variables and integrating was well understood and executed well by the majority of candidates. Most went on to use the correct initial conditions to determine the constant of integration and the majority then went on to derive the given result correctly. The most common errors included incorrect signs used in the integration and incorrect application of the laws of logarithms when attempting to get v^2 in the required form. Part (ii) was often done well with the majority of candidates integrating the correct expression between the required limits; others adopted a work-energy approach. The most common errors with the integrating were to use the net force rather than the resistive force or not consider the lower limit at $x = 0$. Those that applied the work-energy principle usually scored full marks.

Question No. 2 – Variable mass

This question was found to be the most demanding on the paper and very few candidates made significant progress in either part. In part (i) the majority of candidates stated the mass of counterweight correctly as m_1 - $/ t$ but very few could derive the relevant equations of motion for the counterweight and bag correctly. The approach of considering the motion of both parts of the system separately was rarely seen and the majority of candidates attempted to either derive the equation of motion for the whole system from first principles or simply write down the equation of motion for the entire system. The most common errors seen were the inclusion of either incorrect or additional terms involving the quantities m_1, m_2 or / t or the inclusion of an incorrect term involving the velocity of the counterweight (usually present because, when considering the whole system from first principles, the momentum of the sand lost by the counterweight was neglected). Those candidates who did derive the correct differential equation for the velocity of the counterweight usually went on to derive the given result.

Part (ii) was also not well answered by the majority of candidates with many not making any real progress beyond stating the correct relationship between / and m_1. Many candidates did not realise that the maximum velocity of the counterweight was when $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ and simply substituted the t = 10 into their expression for v. Candidates who did find the correct time when the counterweight was at its maximum velocity usually went on to find the correct velocity either exactly or to an acceptable degree of accuracy.

Question No. 3 – Equilibrium

Part (i) was done extremely well by the vast majority of candidates and nearly all handled the necessary calculus and trigonometry accurately.

In part (ii) many candidates did not explicitly show that there was a position of equilibrium when $q = \frac{p}{2}$ but instead assumed that $\frac{\mathrm{d}V}{\mathrm{d}q} = 0$ and attempted to therefore show that $q = \frac{p}{2}$. Most candidates correctly went on to show that the system was in unstable equilibrium at this value of q but not all candidates showed sufficient detail in establishing the given result. A number of candidates made incorrect attempts to use the first derivative test to determine the nature of stability.

The responses to part (iii) were mixed; some excellent succinct solutions were seen, either by taking moments, or applying Hooke's law. Those that adopted the former approach were usually more successful as many of those that took a Hooke's law approach left their answers in terms q. A number of candidates did not realise that the magnitude of the tension in the string and the vertical force at the hinge were equal and therefore they attempted to derive the vertical force from scratch; these attempts were rarely successful. In part (iv), the majority of candidates correctly stated that at the two further positions of equilibrium $mg - \frac{4l}{5}(1 - \sin q) = 0$ but failed to justify accurately the given inequality. Part (v) was rarely done well and many candidates made little progress in this part. Stronger candidates realised that at the other positions of equilibrium the second derivative simplified to $\frac{4}{5} l a \cos^2 q$ but many of these simply stated that this expression was positive in the given interval with the majority failing to consider the case when this expression might be zero.

Question No. 4 - Rotation

Part (a) discriminated quite well and while many candidates scored full marks a number made very little progress beyond stating the moment of inertia of the central wooden cylinder. A number of candidates attempted to derive the moment of inertia of the outer ring of steel from first principles but few did so successfully. Those that correctly found the moment of inertia of the outer ring of steel in terms of the radius of the central cylinder usually went on to find this radius successfully.

Part (b) (i) was done well and many candidates correctly found the kinetic and potential energies of the two blocks in terms of t and went on correctly to derive the given result. Most candidates who attempted a method involving Newton's second law failed to give enough detail to establish convincingly a given answer. Most candidates correctly applied the principle of conservation of energy to derive the stated moment of inertia of the pulley.

Most candidates started part (b) (ii) well with a significant number of them correctly applying the rotational form of Newton's second law to set up the correct differential equation for the angular velocity of the pulley. However, a number of candidates introduced sign errors (mostly that of the resistive couple). While the vast majority of candidates separated the variables of the resulting differential equation and went on to integrate correctly many candidates then tried to find the constant of integration by incorrectly assuming that at $t = 3$ the angular velocity was 0 (rather than the correct value of 24). Those that did apply the correct initial conditions usually went on to derive the correct given result.

The responses to part (b) (iii) were mixed; many candidates either made no response or only attempted part (b) (iii) (A). However, most of those that did attempt part (b) (iii) (A) found the correct value for k at which the pulley continues to rotate with constant angular velocity. It was rare to see the correct set of values for k in either parts (b) (iii) (B) or (C) and even when the correct critical value of 36.75 was seen it was rare to see it used with the correct interval in these two parts.

Unit level raw mark and UMS grade boundaries June 2014 series

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0